

Polyakov Loop Behavior in Non-Extensive SU(2) Lattice Gauge Theory

T. S. Biró

*KFKI Research Institute for Particle and Nuclear Physics,
H-1525 Budapest, P.O.Box 49, Hungary*

Z. Schram

*Department for Theoretical Physics, University of Debrecen,
H-4010 Debrecen, P.O.Box 5, Hungary*

(Dated: January 20, 2013)

Abstract

In order to come closer to a realistic model of high-energy collisions, we simulate SU(2) lattice gauge theory under fluctuating temperature. The fluctuations are Euler-Gamma distributed, leading to a canonical state maximizing the Rényi and Tsallis entropy formulas. This choice conforms to the multiplicity distributions leading to the KNO scaling in high energy experimental spectra. We test the random lattice spacing method numerically by investigating the Polyakov Loop expectation value, known to be a good order parameter for the confinement – deconfinement phase transition in ordinary canonical Monte Carlo methods. The critical coupling (and presumably the temperature) move with the width parameter of the inverse temperature fluctuations towards higher values.

PACS numbers: 65.40.gd, 25.75.Ag

Keywords: non-extensive thermodynamics, lattice gauge theory, Polyakov Loop

I. INTRODUCTION

Lattice gauge theory is up to now the only successful nonperturbative numerical approach to solve physical problems related to the strong interaction. Among the most reknown recent results the prediction of a critical endpoint of the phase transition in QCD became in the forefront of research[1–4]. Also a large scale experimental program, FAIR at GSI, has been initiated, among other goals for studying the interface between quark- and hadronic matter in the CBM experiment [5]. Accelerator experiments, however, do not have a control on thermodynamically relevant parameters, like the temperature and pressure, to such a degree that these could be regarded as having a sharp and constant value during the evolution of the strongly interacting matter. Lattice theoretical simulations on the other hand assume a fixed value for the temperature.

Our aim with the study presented in this paper is to move towards a more flexible scheme: we treat temperature as a random variable, defined not only by its expectation value, but also by a width. In fact the thermodynamically consequent approach to this problem requires that the inverse temperature, $\beta = 1/k_B T$, occurring also as a Lagrange multiplier for the fixed energy constraint by maximizing the entropy, is fixed on the average and then randomized. Such a superstatistical method [6–11] is in accord with recent findings on non-extensive thermodynamics, where the canonical energy distribution is not-exponential, but rather shows an experimentally observed power-law tail [12–17].

In this paper we review basic thermodynamic arguments to relate the temperature to the parameters of a statistical power-law tailed, canonical energy distribution. Following this the superstatistical method is presented, in particular its realization strategy for lattice Monte Carlo simulations. We choose to randomize the timelike to spacelike lattice spacing ratio, $\theta = a_t/a_s$. The most important first task is to check the deconfinement phase transition by observing the Polyakov loop expectation value. These results are presented and discussed. As a main consequence we predict that the deconfinement transition temperature is likely to be higher than determined by fixed- T lattice calculations so far.

II. THERMODYNAMICAL BACKGROUND

Based on arguments regarding the compatibility of general composition rules for the total entropy and energy of composed thermodynamical systems [17], in an extended canonical thermal equilibrium problem the absolute temperature is given by

$$\beta = 1/T = \partial \hat{L}(S)/\partial L(E), \quad (1)$$

with $\hat{L}(S)$ and $L(E)$ being the additive formal logarithms of the respective composition formulas. The formal logarithm maps a general composition law, say $S_{12} = S_1 \oplus S_2$, to the addition by $\hat{L}(S_{12}) = \hat{L}(S_1) + \hat{L}(S_2)$. This construction leads us to maximize $\hat{L}(S) - \beta L(E)$ when looking for canonical energy distributions [16]. The probability distribution, w_i , of states with energy E_i in equilibrium maximizes the formal logarithm of the non-extensive entropy formula with constraints on the average value of the also non-additive energy and the probability normalization:

$$\hat{L}(S)[w_i] - \beta \sum_i w_i L(E_i) - \alpha \sum_i w_i = \max. \quad (2)$$

Here β and α are Lagrange multipliers and it can be proven that $\beta = 1/T$ is related to the thermodynamically valid temperature according to the zeroth law of thermodynamics. Choosing the next to simplest composition formula to the addition, supplemented with a leading second order correction,

$$S_{12} = S_1 + S_2 + \hat{a} S_1 S_2, \quad (3)$$

the additive formal logarithm function is given by

$$\hat{L}(S) = \frac{1}{\hat{a}} \ln(1 + \hat{a} S). \quad (4)$$

This way $\hat{L}(S_{12}) = \hat{L}(S_1) + \hat{L}(S_2)$, indeed. By using the Tsallis entropy formula [18–22],

$$S = \frac{1}{\hat{a}} \sum_i \left(w_i^{1-\hat{a}} - w_i \right), \quad (5)$$

this formal logarithm turns out to be the Rényi entropy [23, 24]

$$\hat{L}(S) = \frac{1}{\hat{a}} \ln \sum_i w_i^{1-\hat{a}}. \quad (6)$$

It is customary to use the parameter, $q = 1 - \hat{a}$. The above power-law tailed form of energy distribution can be fitted to experimentally observed particle spectra, and this way

a numerical value for the parameter \hat{a} can be obtained. The $\hat{a} = 0$ ($q = 1$) case recovers the classical Boltzmann-Gibbs-Shannon (BGS) formula [25–28]

$$S_{BG} = \sum_i -w_i \ln w_i. \quad (7)$$

According to this the quantity $\hat{L}(S)$ is to be maximized with constraints. Identifying the analogous formal logarithm for leading order non-additive energy composition, $E_{12} = E_1 + E_2 + aE_1E_2$, as

$$L(E) = \frac{1}{a} \ln(1 + aE), \quad (8)$$

one considers

$$\frac{1}{\hat{a}} \ln \sum_i w_i^{1-\hat{a}} - \beta \sum_i w_i \frac{1}{a} \ln(1 + aE_i) - \alpha \sum_i w_i = \max. \quad (9)$$

The maximum is achieved by the canonical probability distribution

$$w_i = A (b(\alpha + \beta L_i))^{-\frac{1}{\hat{a}}} \quad (10)$$

with

$$L_i = \frac{1}{a} \ln(1 + aE_i), \quad A = e^{-\hat{L}(S)}, \quad b = \frac{\hat{a}}{1 - \hat{a}}. \quad (11)$$

Then the normalization, the average and the definition of the entropy lead to the condition

$$1 = b\alpha + b\beta \langle L \rangle. \quad (12)$$

Finally the equilibrium distribution simplifies to

$$w_i = \frac{1}{Z} (1 + \hat{a}\hat{\beta}L_i)^{-1/\hat{a}} \quad (13)$$

with L_i given in eq.(11). Here we have introduced the following shorthand notations:

$$Z = \frac{1}{A} (1 - b\beta \langle L \rangle)^{\frac{1}{\hat{a}}}, \quad \hat{\beta} = \frac{\beta}{1 - \hat{a}(1 + \beta \langle L \rangle)}. \quad (14)$$

We should keep in mind that the reciprocal temperature, distinguished by the Zeroth Law, is the Lagrange multiplier β . This is reflected well by the whole formalism, because the usual thermodynamic relations are valid.

It is particularly interesting to consider now cases, when only one of the two quantities is composed by non-additive rules. In the limit of additive entropy but non-additive energy ($\hat{a} \rightarrow 0$) the canonical distribution approaches

$$w_i = \frac{1}{Z_0} (1 + aE_i)^{-\beta/a}, \quad \text{where} \quad \ln Z_0 = S_{BG} - \beta \langle E \rangle. \quad (15)$$

Here S_{BG} is the Boltzmann-Gibbs-Shannon entropy (cf. eq.7). For non-additive entropy and additive energy on the other hand a similar, but differently parametrized power-law tailed distribution emerges:

$$w_i = \frac{1}{Z} \left(1 + \hat{\beta} \hat{a} E_i\right)^{-1/\hat{a}}, \quad (16)$$

with

$$\hat{\beta} = \frac{\beta}{1 - \hat{a}(1 + \beta \langle E \rangle)}. \quad (17)$$

The latter relation can be transformed into a more suggestive form by using $q = 1 - \hat{a}$ and the temperature parameters $T = 1/\beta$ and $\hat{T} = 1/\hat{\beta}$:

$$T = \frac{1}{q} \hat{T} + \left(\frac{1}{q} - 1\right) \langle E \rangle. \quad (18)$$

By using the distribution given in eq.(16), the expectation value of the energy, $\langle E \rangle$, is directly given as a function of \hat{T} and $\hat{a} = 1 - q$.

III. SUPERSTATISTICAL MONTE CARLO METHOD

In either case discussed in the previous section, the generalized canonical distribution of the different energy states in a system in thermal equilibrium with non-additive composition rules is given by a formula

$$w_i = \frac{1}{Z_{TS}} \left(1 + \frac{\beta E_i}{c}\right)^{-c}. \quad (19)$$

In the $c \rightarrow \infty$ limit this formula coincides with the familiar Gibbs factor:

$$\lim_{c \rightarrow \infty} w_i = \frac{1}{Z_G} \exp(-\beta E_i). \quad (20)$$

The quantity $q = 1 - 1/c$ is called the Tsallis index. Here $c = \beta/a$ and β is in fact the inverse absolute temperature for the energy non-additivity case; for the entropy non-additivity on the other hand β has to be replaced by $\hat{\beta}$ and c by $1/\hat{a}$ as it was explained in the previous section. The thermodynamic temperature in the latter case, according to the Zeroth Law, can be obtained by using eq.(18).

The Tsallis distribution weight factor, w_i , on the other hand can be obtained as an integral of Gibbs factors over the Gamma distribution [29, 30],

$$w_i = \frac{1}{Z_{TS}} \int_0^\infty d\theta w_c(\theta) \exp(-\theta \beta E_i), \quad (21)$$

with

$$w_c(\theta) = \frac{c^c}{\Gamma(c)} \theta^{c-1} e^{-c\theta}. \quad (22)$$

$\Gamma(c) = (c-1)!$ for integer c is Euler's Gamma function. By its definition the integral of $w_c(\theta)$ is normalized to one. This approach is a particular case of the so called *superstatistics* [6, 8].

Based on this, any canonical Gibbs expectation value, if known as a function of β , can be converted into the corresponding expectation values with the power-law tailed canonical energy distribution. The respective partition functions, Z_G and Z_{TS} ensure the normalization of the w_i probabilities, $\sum_i w_i = 1$. They are related to each other:

$$Z_{TS}(\beta) = \sum_i \int_0^\infty d\theta w_c(\theta) \exp(-\theta\beta E_i) = \int_0^\infty d\theta w_c(\theta) Z_G(\theta\beta). \quad (23)$$

The above formula can be interpreted as averaging over different $\theta\beta$ -valued Gibbs simulations. The averaging is understood in the partition sum, meaning that the weighting 'Boltzmann'-factor is also fluctuating. It assumes that the underlying process of mixing different inverse temperatures is much faster than the averaging itself.

The question arises, which strategy is the best to follow in order to perform lattice field theory simulations with power-law tailed statistics instead of the Gibbs one. Neither the ensemble of different β values (Euclidean timelike lattice sizes), nor the re-sampling of the traditional, Gibbs distributed configurations is practicable in a naive way. The N_t lattice sizes are limited to a small number of integer values – hence the good coverage of a Gamma distribution with an arbitrary real c value is questionable. The already produced configuration ensembles were selected by a Monte Carlo process according to the Gibbs distribution with the original lattice action; there is no guarantee that the re-weighting procedure (which includes part of the weight factors in the operator expressions for observables) is really convergent (i.e. does not contain parts growing exponentially or worse). We choose another strategy: we use θ values selected as random deviates from an Euler-Gamma distribution during the Monte Carlo statistics.

The lattice simulation incorporates the physical temperature by the period length in the Euclidean time direction: $\beta = N_t a_t$. Due to the restriction to a few integer values of N_t , we simulate the Gamma distribution of the physical $\beta = 1/T$ values by a Gamma distribution of the timelike link lengths, a_t . We assume that its mean value is equal to the spacelike lattice spacing, a_s . Then the ratio $\theta = a_t/a_s$ follows a normalized Gamma distribution with

the mean value 1 and a width of $1/\sqrt{c}$. (In the view of ZEUS e^+e^- data $c \approx 5.8 \pm 0.5$, the width is about 40 per cent.) In our numerical calculations we apply the value $c = 5.5$.

For calculating expectation values in field theory a generating functional based on the Legendre transform of Z is used. Our starting assumption is the formula (23) with

$$Z_G[\theta\beta] = \int \mathcal{D}U e^{-S[U,\theta]}. \quad (24)$$

Since we simulate the canonical power-law distribution by a lattice with fluctuating asymmetry ratio, there are two limiting strategies to execute the Legendre transformation: i) in the *annealing* scenario the lattice fluctuates slowly and one considers first summations over field configurations, in the ii) *quenched* scenario on the contrary, the lattice fluctuations are fast, form an effective action (virtually re-weighting the occurrence probability of a field configuration), and the summation over possible field configuration is the slower process performing the second (i.e. the path-) integral. In this paper we investigate numerically the general case when one may choose when a new value for θ is taken. The frequency of these fluctuations may go from one in each Metropolis step for the field configurations to one in the whole Monte Carlo process (the latter being the traditional method). Our results presented in the next section belong to a choice of 5 field updates for the whole lattice before choosing a new θ . This peculiar value was controlled by a series of simulations and proved to be sufficient for a close equilibration to a given, momentary temperature [31].

The effect of θ fluctuation is an effective weight for field configurations, which may depend on a scaling power according to the time (or energy) dimension of the operator under study. In general we consider the Tsallis expectation value of an observable $\hat{A}[U]$ over lattice field configurations U . \hat{A} may include the timelike link length, say with the power v : $\hat{A} = \theta^v A$. The Tsallis expectation value then is an average over all possible a_t link lengths according to a Gamma distribution of $\theta = a_t/a_s$. We obtain:

$$\langle A \rangle_{TS} = \frac{1}{Z_{TS}} \frac{c^c}{\Gamma(c)} \int d\theta \theta^{c-1} e^{-c\theta} \int \mathcal{D}U A[U] \theta^v e^{-S[\theta,U]} \quad (25)$$

with

$$Z_{TS} = \frac{c^c}{\Gamma(c)} \int d\theta \theta^{c-1} e^{-c\theta} \int \mathcal{D}U e^{-S[\theta,U]}. \quad (26)$$

The θ dependence of the lattice gauge action is known for long: due to the time derivatives of vector potential in the expression of electric fields, the "kinetic" part scales like $a_t a_s^3 / (a_t^2 a_s^2) =$

a_s/a_t , and the magnetic ("potential") part like $a_t a_s^3/(a_s^2 a_t^2) = a_t/a_s$ [34]. This leads to the following expression for the general lattice action:

$$S[\theta, U] = a \theta + b/\theta, \quad (27)$$

here $a = S_{ss}[U]$ contains space-space oriented plaquettes and $b = S_{ts}[U]$ contains time-space oriented plaquettes. The simulation runs in lattice units anyway, so actually the U configurations are selected according to weights containing a and b . In the $c \rightarrow \infty$ limit the scaled Gamma distribution approximates $\delta(\theta - 1)$, (its width narrows extremely, while its integral is normalized to one), and one gets back the traditional lattice action $S = a + b$, and the traditional averages. For finite c , one can exchange the θ integration and the configuration sum (path integral) and obtains exactly the power-law-weighted expression.

IV. STATISTICS OF POLYAKOV LOOPS

Before discussing our results for the SU(2) pure gauge lattice field simulation using Euler-Gamma distributed timelike lattice spacing (and simulating this way a fluctuating inverse temperature to leading order in non-extensive thermodynamics), let us present a figure about the numerical quality of this randomization. In Fig.1 the evolution process and the frequency distribution of the θ values are shown for the reference run with $c = 1024.0$ and for the investigated case with $c = 5.5$. We have chosen a new value for the asymmetry ratio θ in each 5-th Monte Carlo update – in order to leave some time for the relaxation of the field to its thermal state at each instantaneous $\beta\theta$ inverse temperature. In the figure only each 5-th value is shown. The Monte Carlo simulations were done at the coupling $4/g^2 = 2.40$ for this particular statistics with the Metropolis method.

Our reference case, thought to be close to the $c = \infty$ traditional system, is specified by $c = 1024$. The re-fit to the distribution of effectively used values after 20000 draws from the Euler-Gamma distribution by a numerical subroutine was done by the statistics tool "gretl". In the special case of our random weighting one expects an Euler-Gamma distribution with reciprocal $\alpha = c$ and $\beta = 1/c$ parameters. On the basis of a sample of 20000 θ values we achieved a reconstruction of $\alpha = 1009.8$ and $1/\beta = 1010.1$. Similarly for $c = 5.5$ we obtained $\alpha = 5.5179$ and $1/\beta = 5.5255$.

Now let us turn to the discussion of the behavior of the order parameter of the confinement

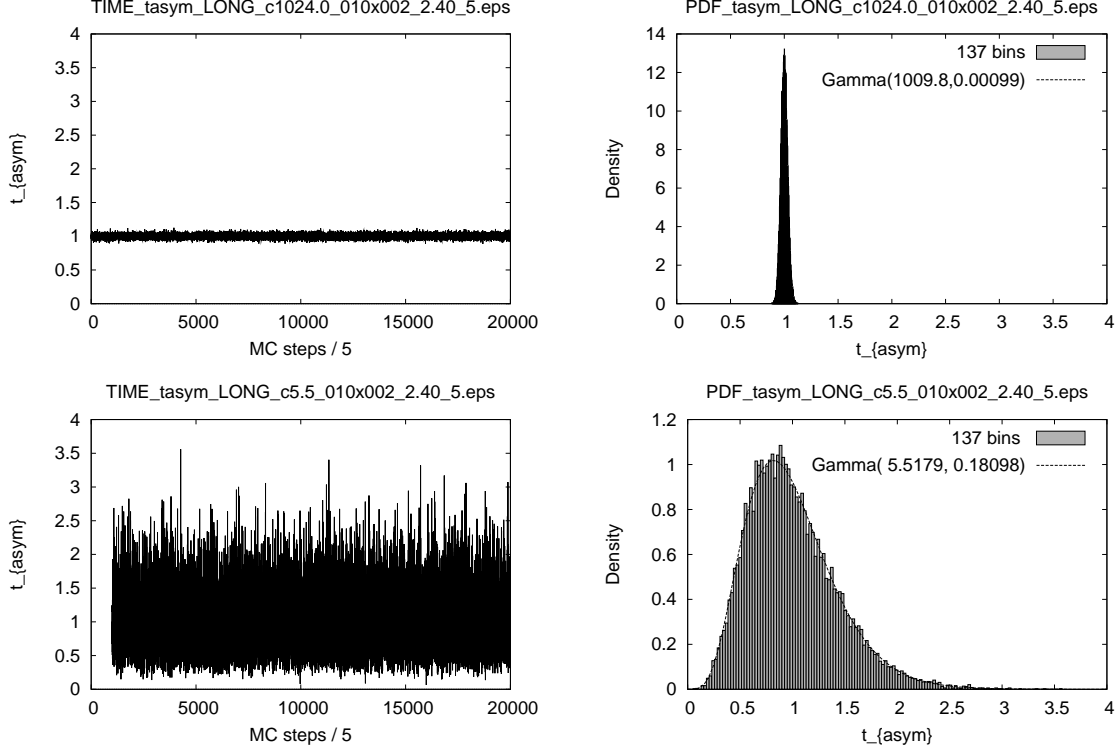


FIG. 1: The Monte Carlo evolution and distribution of $t_{\text{asym}} = a_t/a_s$ for the coupling $4/g^2 = 2.40$. The random deviates for the results shown in the upper row are thrown with the parameters $\alpha = c = 1024.0$ and $\beta = 1/c = 0.000977$. The re-fit by gretl gave $\alpha = 1009.8$ and $\beta = 0.000990$. The same parameters in the lower row are $\alpha = c = 5.5$ and $\beta = 1/c = 0.181818$. The re-fit by gretl gave $\alpha = 5.5179$ and $\beta = 0.18098$.

– deconfinement phase transition. The Polyakov Loop is calculated by taking the trace of the product of gauge group elements on timelike links closing a loop due to the periodic boundary condition:

$$P(x) = \text{Tr} \prod_{t=1}^{N_t} U_t(t, x). \quad (28)$$

The traditional order parameter of the phase transition is the expectation value of the volume averages for each lattice field-configuration during the Monte Carlo process. For the gauge group $SU(2)$ this quantity is real:

$$\Re P = \Re \sum_x P(x). \quad (29)$$

In our present investigations the characteristic width parameter of $1/T$ -fluctuations is $c =$

5.5, corresponding to a relative width of $1/\sqrt{c} \approx 0.43$. As a reference the $c = 1024.0$ case is taken – here the relative width is about $1/\sqrt{c} = 1/32 \approx 0.03$.

The plots in Fig.2 show the fluctuations of the order parameter $\Re P$ for the reference runs with $c = 1024.0$. The fluctuating values as a function of the Monte Carlo step are plotted on the left hand side, while their probability distributions on the right hand side. The values for the inverse coupling include both the confinement and deconfinement phases.

By producing these results we took five consecutive Metropolis sweeps over the whole 4-dimensional $10^3 \times 2$ lattice while keeping the asymmetry value $\theta = a_t/a_s$ constant. Then a new θ was chosen as a random deviate from an Euler-Gamma distribution. Only these 5-th values are plotted and counted for obtaining expectation values. The probability distributions of these values were determined by using the statistics software tool "gretl". Hereby the first 5000 configurations were sometimes taken out from the samples, consisting of 100000 lattice configurations each, this did not change expectation values appreciably. For the statistical evaluation only each 5-th configuration was selected, being fairly independent of each other in the evolution governed by the Metropolis algorithm and certainly belonging to different θ values. The frequency distributions reflect cleanly when several $\Re P$ expectation values are occurring during the Monte Carlo evolution, by several maxima. This is the case near to the phase transition point.

Similar pictures from Monte Carlo simulations with fluctuating inverse temperature using the parameter $c = 5.5$ are plotted in the figures 3 – 7. Here the effect of the width in the possible temperature values is clearly seen in the larger fluctuations of the order parameter compared to the reference case $c = 1024.0$ at the same coupling. Also the critical inverse coupling strength moves towards higher values for $c = 5.5$. In Fig.5 we zoom to the neighborhood of the critical coupling: The distribution of the $\Re P$ values are characteristically wide. In the third row, at $4/g^2 = 2.14$, the distribution of possible values is almost flat between -1 and 1 . (Due to the $SU(2)$ trace normalization, as we use it, the maximal absolute value of the order parameter is 2 .) The intermittent behavior between positive and negative values of $\Re P$, a sure sign of the restoration of the center symmetry Z_2 , can be caught until the value $4/g^2 = 2.20$, as it can be inspected in Fig.6. For even higher inverse coupling strength the observational sample is too short to observe this effect.

How to estimate the critical coupling for the appearance of the nonzero order parameter? The method closest to the traditional one[32] is to take the average value over the statis-

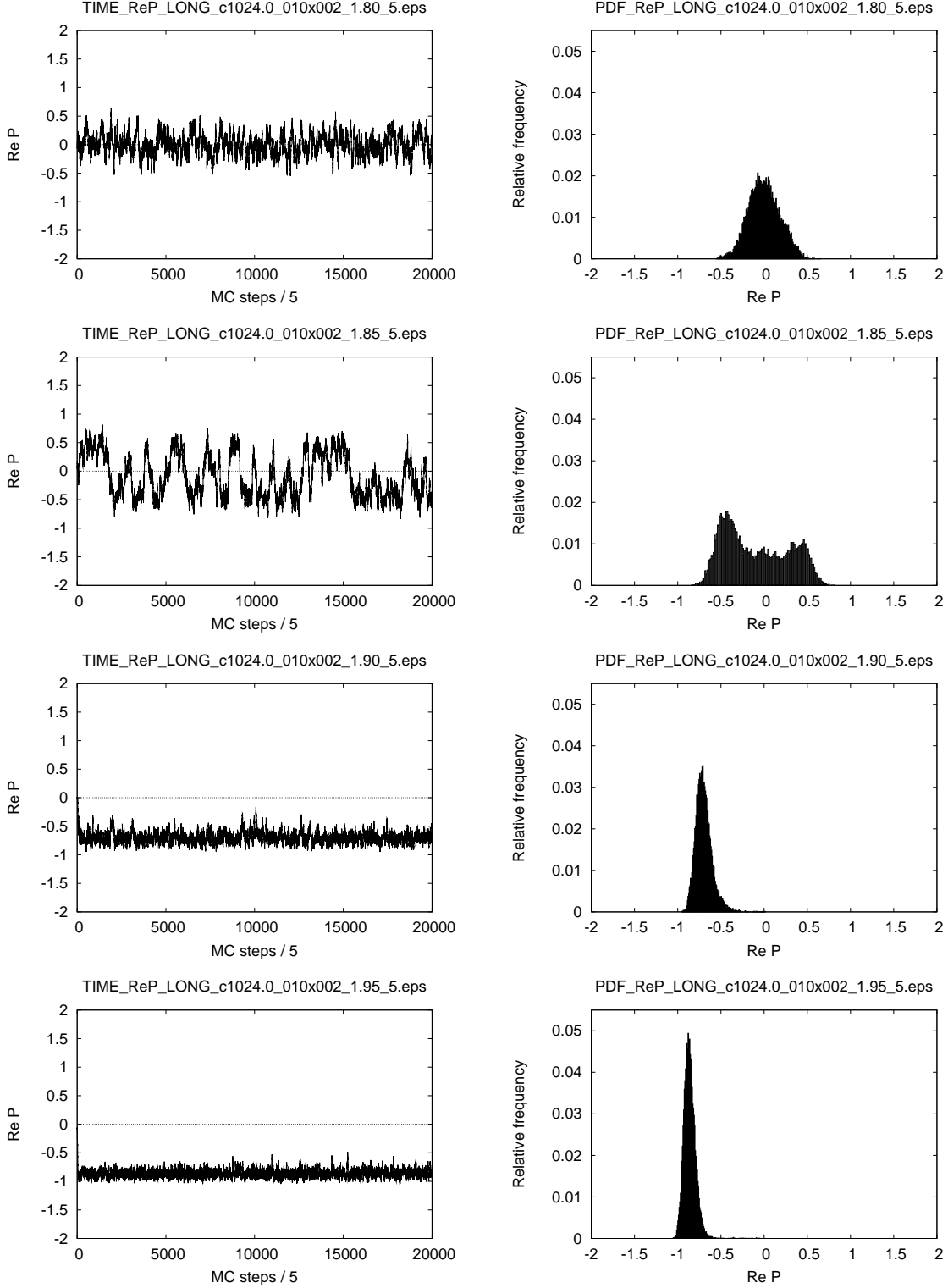


FIG. 2: The Monte Carlo evolution and distribution of $\text{Re } P$ for the couplings $4/g^2 = 1.80, 1.85, 1.90$ and 1.95 using $c = 1024.0$ from the top to the bottom. This reference pictures show a nearly-traditional confinement – deconfinement phase transition for the SU(2) Yang-Mills system. Note the small width of the order parameter distribution.

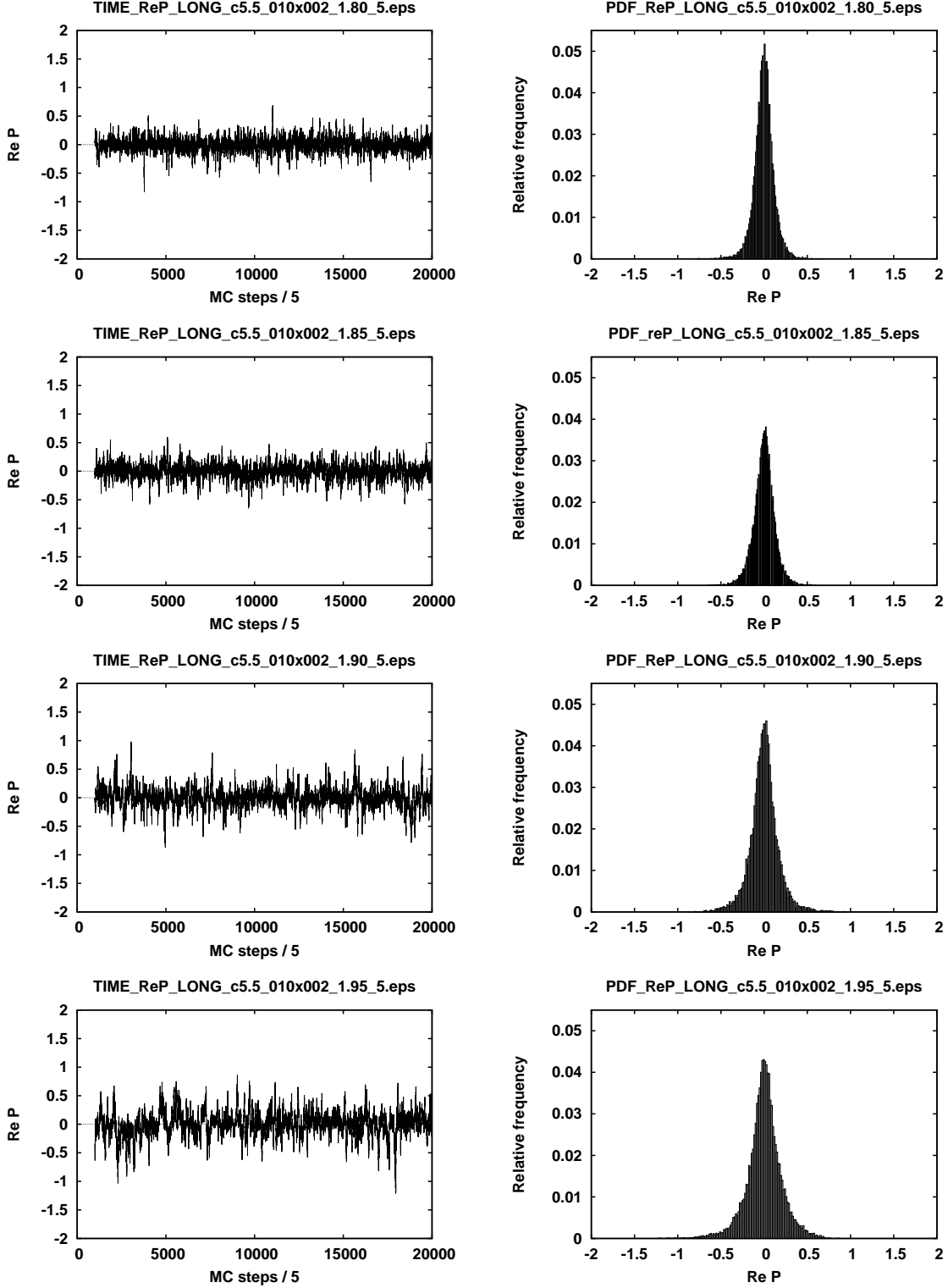


FIG. 3: The Monte Carlo evolution and distribution of $\text{Re } P$ for the couplings $4/g^2 = 1.80, 1.85, 1.90$ and 1.95 using $c = 5.5$ from the top to the bottom. Confinement phase.

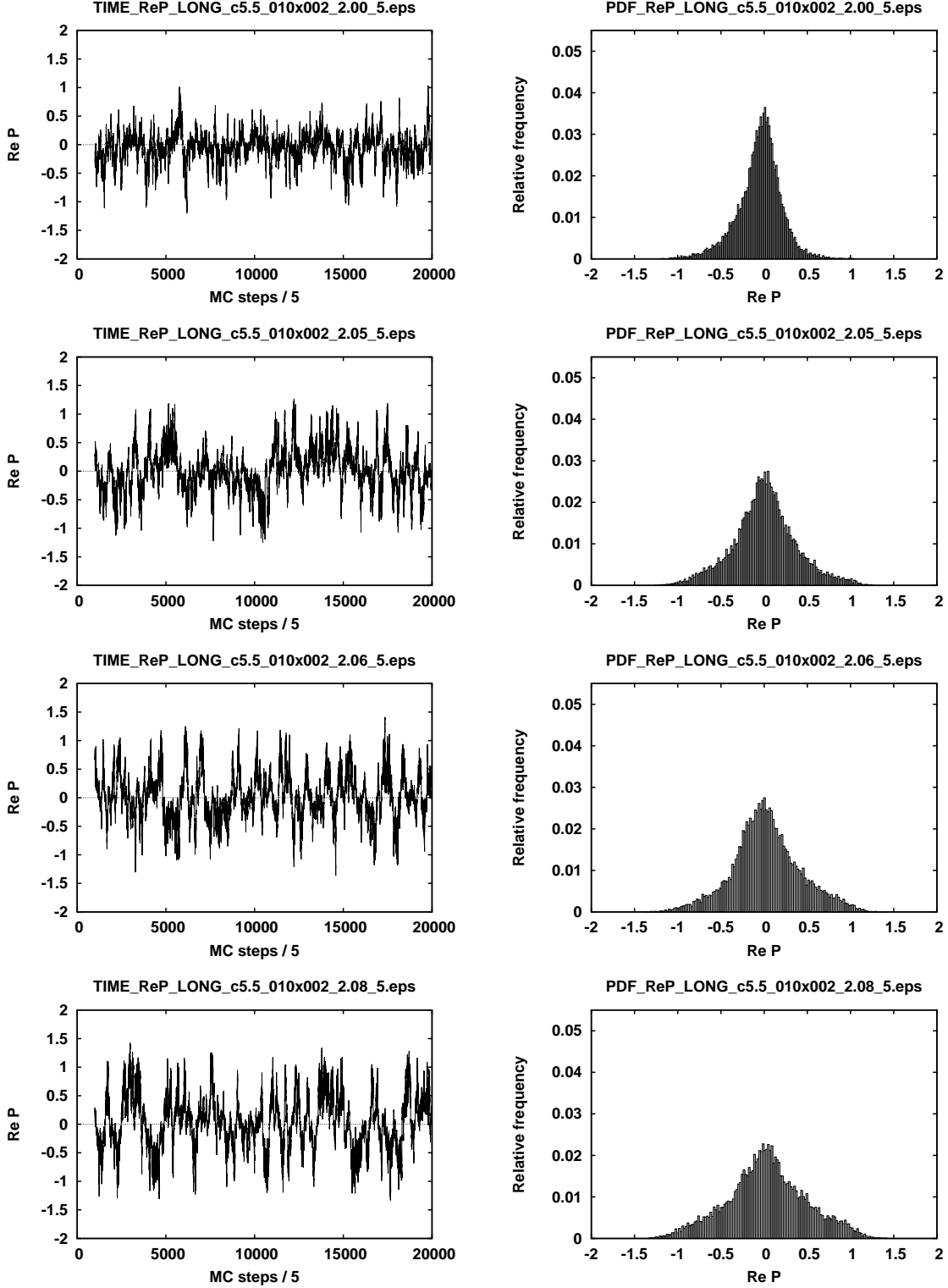


FIG. 4: The Monte Carlo evolution and distribution of $\Re P$ for the couplings $4/g^2 = 2.00, 2.05, 2.06$ and $2.08i$ using $c = 5.5$ from the top to the bottom. These couplings are nearly critical.

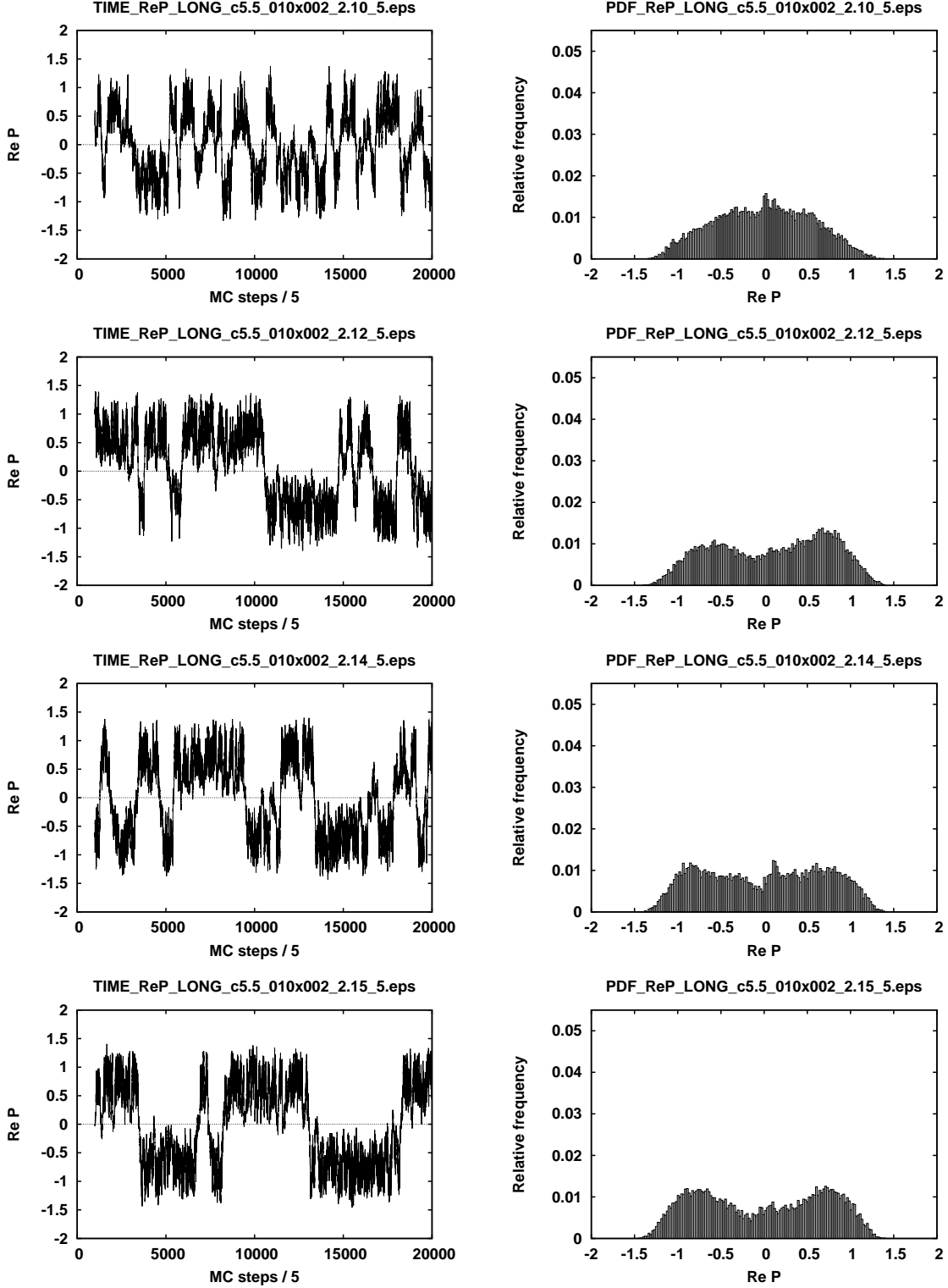


FIG. 5: The Monte Carlo evolution and distribution of $\text{Re } P$ for the couplings $4/g^2 = 2.10, 2.12, 2.14$ and 2.15 using $c = 5.5$ from the top to the bottom. Here the two-peak distribution develops, the deconfinement sets in.

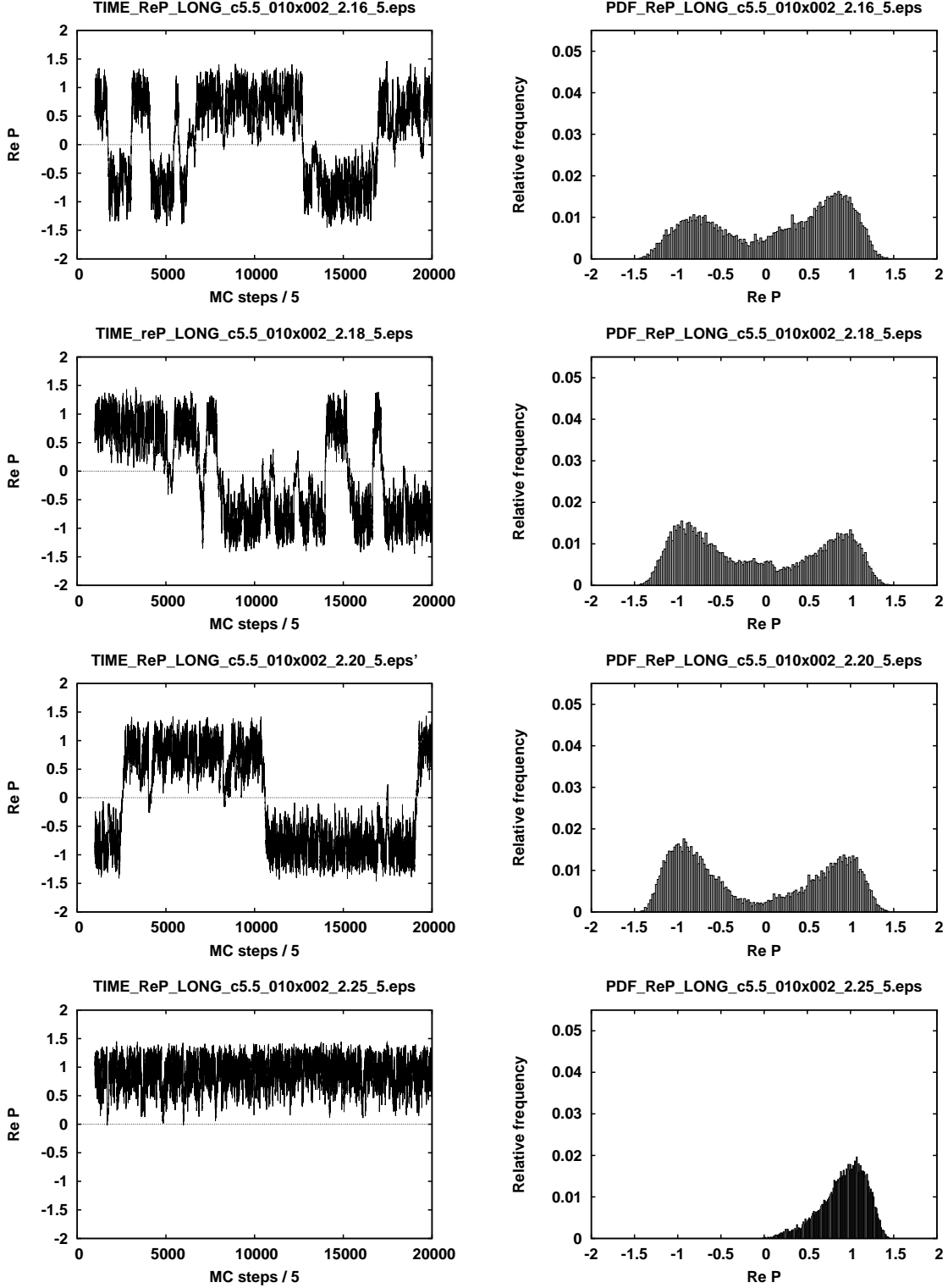


FIG. 6: The Monte Carlo evolution and distribution of $\text{Re } P$ for the couplings $4/g^2 = 2.16, 2.18, 2.20$ and 2.25 using $c = 5.5$ from the top to the bottom. By these couplings we dwell into the deconfinement regime.

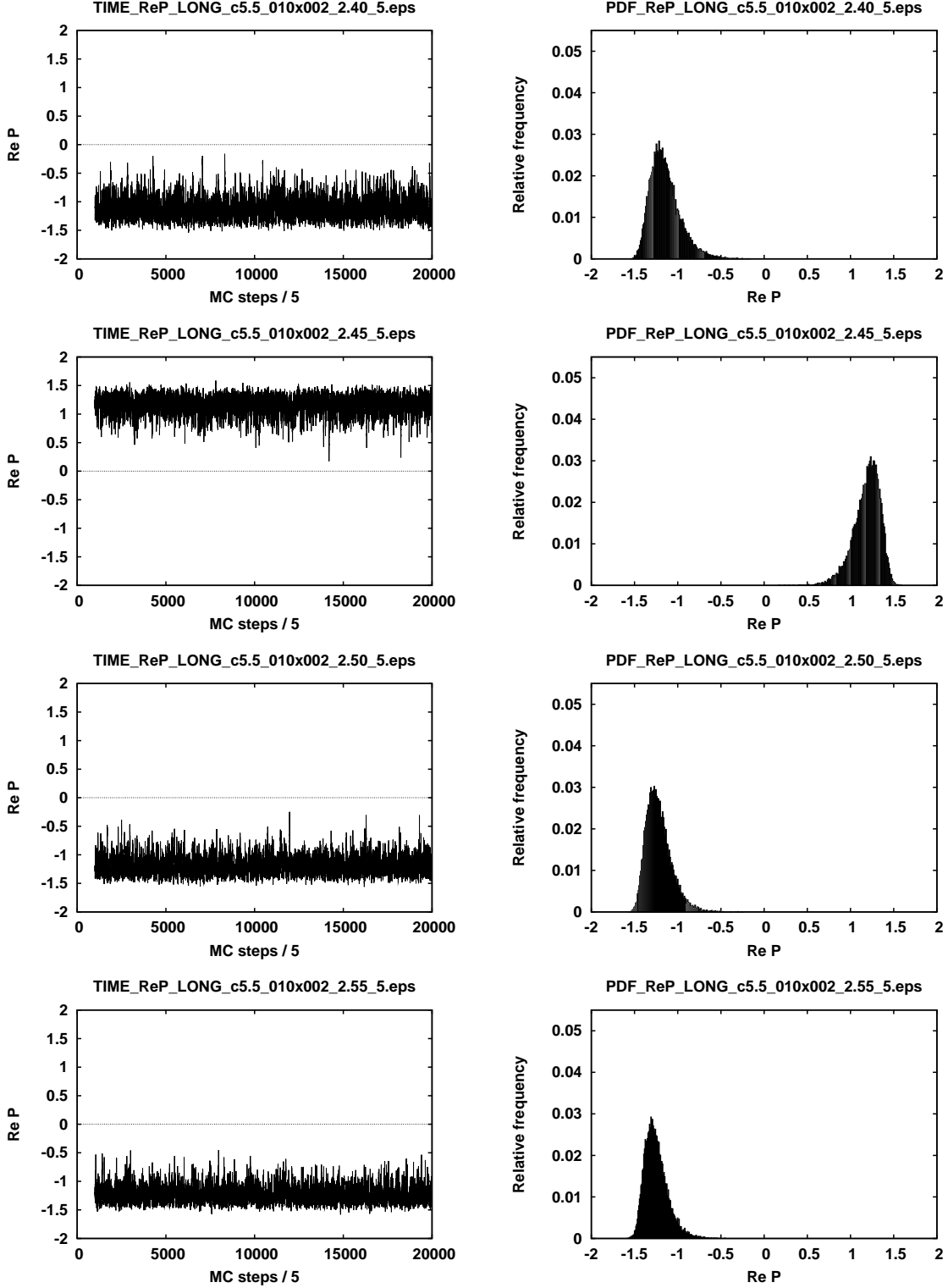


FIG. 7: The Monte Carlo evolution and distribution of $\Re P$ for the couplings $4/g^2 = 2.40, 2.45, 2.50$ and 2.55 using $c = 5.5$ from the top to the bottom. For these couplings only one symmetry breaking maximum occurs representing a well-developed deconfinement phase.

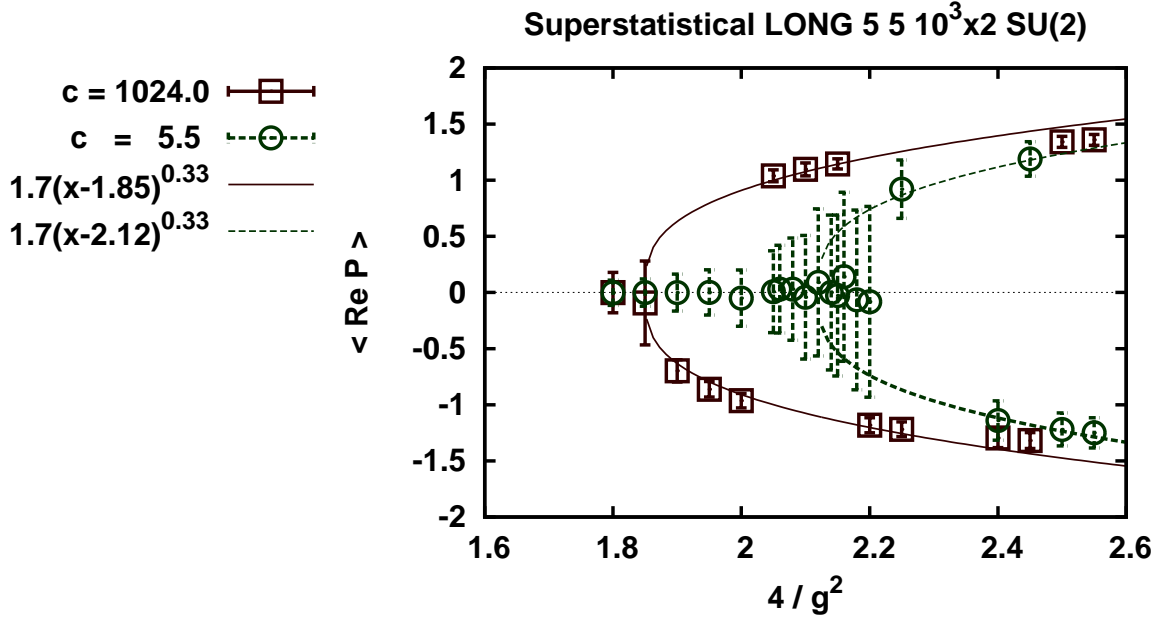


FIG. 8: Results on Polyakov Loop spatial average expectation values in long runs (100.000 Monte Carlo steps, each 5-th kept) on $10^3 \times 2$ lattices at $c = 5.5$ (red squares) and at $c = 1024.0$ (green circles). The Gaussian widths are indicated by error bars. The transition point, i.e. the critical coupling strength, $x = 4/g_c^2$, is estimated by a functional fit, $\Re P \sim (x - x_c)^{1/3}$.

tics. In Fig.8 we plot $\langle \Re P \rangle$ over the longer Monte Carlo runs presented above with their distribution. There is a characteristic difference between the $c = 5.5$ and the $c = 1024.0$ cases. A possible fit to the average values is given by a fractional power; it seems that a $1/3$ power-law behavior describes the critical scaling well. Of course, on the basis of the present data a square root behaviour also cannot be excluded. The obtained positions of the critical couplings differ: $4/g_c^2 \approx 1.85$ for $c = 1024.0$ while $4/g_c^2 \approx 2.12$ for $c = 5.5$.

For drawing conclusions relevant to the physics the inverse lattice couplings have to be related to temperatures. Figure 10 presents T/T_c ratios versus the inverse coupling, $4/g^2$ for $N_t = 2$ lattices, based on data for critical couplings on different N_t -sized lattices [33]. Although those simulations were carried out without temperature fluctuations, i.e. taking $c = \infty$, we use them as a first estimate for the temperature – coupling correspondence. The

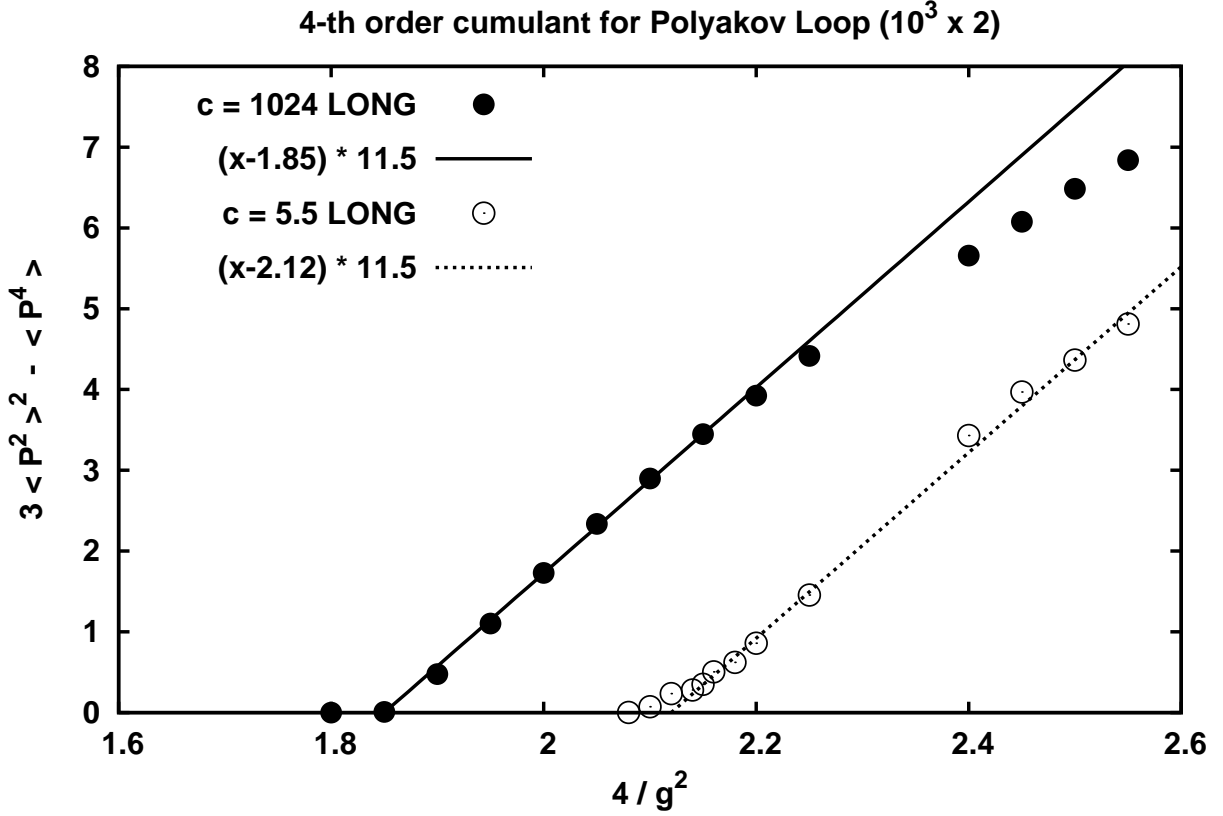


FIG. 9: Fourth order cumulants of the Polyakov Loop spatial average in long runs (100.000 Monte Carlo steps, each 5-th kept) on $10^3 \times 2$ lattices at $c = 5.5$ (full circles) and at $c = 1024.0$ (open circles). The critical coupling strength, $x = 4/g_c^2$, is obtained by a linear fit to the smaller nonzero values.

critical coupling in our calculation for $c = 1024.0$ is close to the result obtained previously on same sized ($N_t = 2$) lattices. The critical coupling at $c = 5.5$ – following the $c = \infty$ line of constant physics – corresponds on the other hand to a temperature which is 1.3 times higher than the usual value.

V. CONCLUSION

1. For $c = 5.5$ (a realistic value from p_T spectra) the critical coupling at the deconfinement phase transition shifts towards higher values. To this value an increase of the deconfinement temperature is obtained at $T_c(5.5) \approx 1.3T_c(1024) \approx 1.3T_c(c = \infty)$.
2. Aiming at the same $1/T$ value for the simulation, i.e. $\langle \theta \rangle = 1$, the temperature is

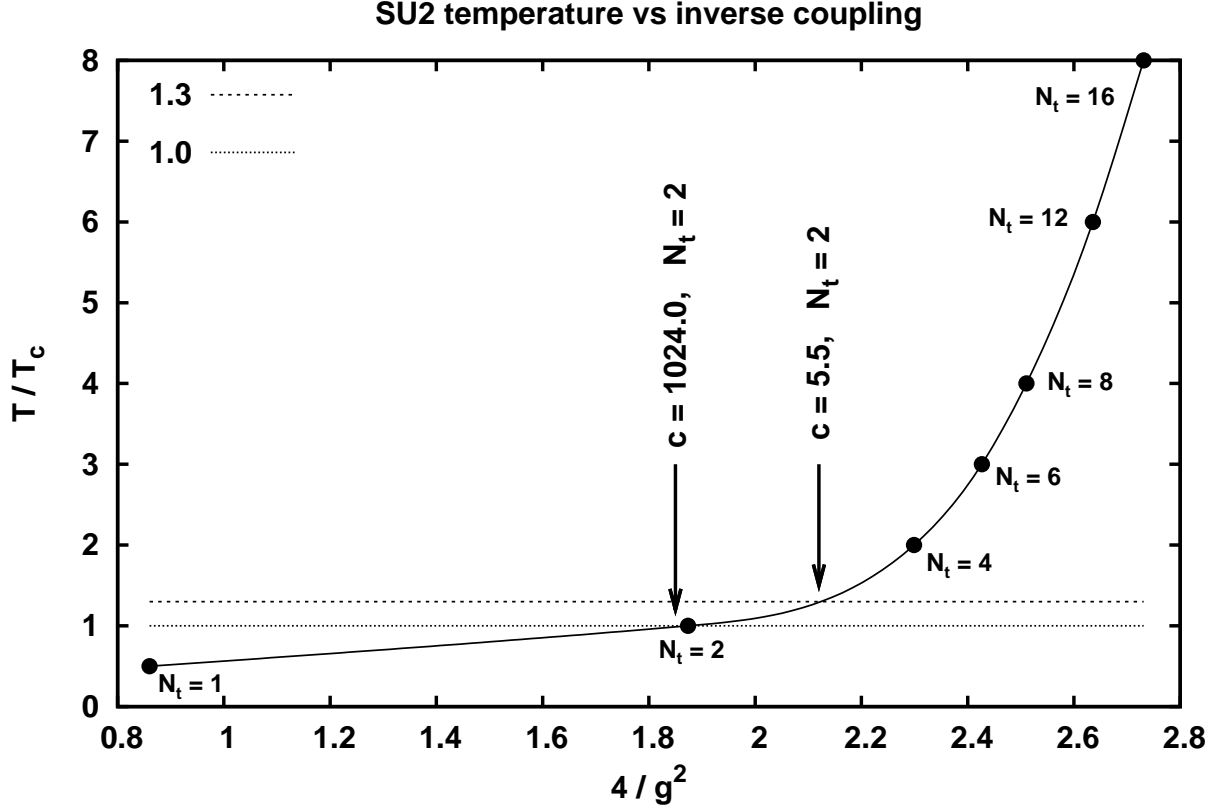


FIG. 10: Deconfinement temperatures based on [33] vs inverse coupling strength obtained from different size lattice simulations (N_t values are indicated on the plot). The arrows point to our findings of critical temperatures with $N_t = 2$ for $c = 1024.0$ and $c = 5.5$, respectively. The corresponding horizontal lines are drawn at 1.00 and 1.30 with respect to the $c = \infty$ case.

expected to make an increase of about 20 per cent due to $\langle 1/\theta \rangle = c/(c-1) \approx 1.22$. This shows the same trend as obtained by the Monte Carlo simulations, but not its whole magnitude.

3. We obtained, assuming the traditional scaling dependence between coupling and physical temperature, an increase of 15 per cent in $4/g_c^2$ leading to about an increase of 30 per cent in T_c . The dynamical effect is definitely larger than the trivial statistical factor of 1.22.
4. Therefore experiments aiming at producing quark matter under circumstances characteristic to high energy collisions should consider the possibility of an about 30 per cent higher T_c than predicted by traditional Monte Carlo lattice calculations. A pos-

sible measurement of the value of the width parameter c can be achieved by analyzing event-by-event spectra.

These preliminary conclusions are based on a comparison with the $c = \infty$ traditional results. In future works we aim to explore the $T/T_c - 4/g^2$ curve and possibly the renormalization of physical quantities under the condition of fluctuating temperature with finite c values.

Acknowledgment

This work has been supported by the Hungarian National Science Fund, OTKA (K68108) and by the TÁMOP 4.2.1/B-09/1/KONV-2010-0007 project co-financed by the European Union and the European Social Fund. Partial support from the Helmholtz International Center (HIC) for FAIR within framework of the Landes-Offensive zur Entwicklung Wirtschaftlich-Ökonomischer Exzellenz (LOEWE) launched by the State of Hesse, Germany. Discussions with Prof. B. Müller and A. Jakovác are gratefully acknowledged.

-
- [1] Fodor Z. Katz S.D., JHEP**0203**(2002)014.
 - [2] Fodor Z. Katz S.D., JHEP**0404**(2004)050.
 - [3] Cheng M. Christ N.H. Datta S. van der Heide J. Jung C. Karsch F. Kaczmarek O. Laermann E. Mawhinney R.D. Miao C. Petreczky P. Petrov K. Schmidt C. Soeldner W. Umeda T., Phys. Rev. D**77**(2008)014511.
 - [4] Cheng M. Christ N.H. Datta S. van der Heide J. Jung C. Karsch F. Kaczmarek O. Laermann E. Mawhinney R.D. Miao C. Petreczky P. Petrov K. Schmidt C. Soeldner W. Umeda T., Phys. Rev. D**74**(2006)054507.
 - [5] CBM Progress report 2010, <https://www.gsi.de/documents/DOC-2011-Mar-235-1.pdf>.
 - [6] Beck C. Cohen E.G.D., Physica A**322**(2003)267; *ibid***344**(2004)393.
 - [7] Touchette H. Beck C., Phys. Rev. E**71**(2005)016131.
 - [8] Abe S. Beck C. Cohen G.D., Phys. Rev. E**76**(2007)031102.
 - [9] Jizba P. Kleinert H., Phys. Rev. E**78**(2008)031122.
 - [10] Van der Straaten E. Beck C., Phys. Rev. E**78**(2008)051101.

- [11] Beck C., Eur. Phys. J. A**40**(2009)267.
- [12] Biró T.S. Jakovac A., Phys. Rev. Lett.**94**(2005)132302.
- [13] Biró T.S. Ürmösy K., J. Phys. G**36**(2009)064044.
- [14] Biró T.S., Eur. Phys. Lett.**84**(2008)56003.
- [15] Biró T.S., Purcsel G. Ürmösy K., Eur. Phys. J. A**40**(2009)325.
- [16] Biró T.S., *Is There a Temperature? – Conceptual Challenges at High Energy, Acceleration and Complexity*, Springer New York, 2011.
- [17] Biró T.S. and Ván P., *Zeroth-Law compatibility of non-additive thermodynamics*, arXiv 1101.0023, 2011.
- [18] Tsallis C., *Introduction to Non-Extensive Statistical Mechanics: Approaching a Complex World*, Springer, 2009.
- [19] Tsallis C., J. Stat. Phys.**52**(1988)50.
- [20] Tsallis C., Physica A**221**(1995)277.
- [21] Tsallis C., Braz. J. Phys.**29**(1999)1.
- [22] Tsallis C., Eur. Phys. J. A**40**(2009)257.
- [23] Rényi A., Acta Math. Acad. Sci. Hung.**10**(1959)193.
- [24] Renyi A., *Probability Theory*, North Holland, Amsterdam, 1970.
- [25] Shannon C. E., Bell Syst. Tech. J.**27**(1948)379; *ibid.* **27**(1948)623.
- [26] Jaynes E. T., Phys. Rev.**160**(1957)171; *ibid.* **168**(1957)620.
- [27] Daróczy Z. Aczél J., *On Measures of Information and their Characterization*, Academic Press, New York, 1975.
- [28] Havrda J. H. Charvat F., Kybernetika**3**(1967)30.
- [29] Biro T.S. Purcsel G. Gyorgyi G. Jakovac A. Schram Z., Nucl. Phys. A**774**(2006)845.
- [30] Biro T.S. Schram Z., J. Phys. G**37**(2010)094027.
- [31] Biró T.S. Schram Z., Acta Physica Debrecina**XLIV**(2010)130.
- [32] Kuti J. Polónyi J. Szlachányi K., Phys. Lett. B**98**(1981)199.
- [33] Velytsky A., *Finite temperature SU(2) gauge theory: critical coupling and universality class*, Int. J. Mod. Phys. C**19**(2008)1079.
- [34] This generalizes to all lattice field actions: kinetic and mass terms scale like $1/\theta$, potential terms like θ .